Interest Rate Risk Modeling

The Fixed Income Valuation Course

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- **Interest Rate Risk Modeling : The Fixed Income Valuation Course.** Sanjay K. Nawalkha, Gloria M. Soto, Natalia A. Beliaeva, 2005, Wiley Finance.
  - **Chapter 1: Interest Rate Risk Modeling: An Overview**

- Goals:
  - Introduce the many types of risks financial institutions and other market participants manage.
  - Detail Interest Rate Risk: What it is? How to manage it?
  - Explain the main models of interest rate and discuss some examples.

Chapter 1: Interest Rate Risk Modeling: An Overview

- Introduction
- Duration and Convexity Models
- M-Absolute and M-Square Models
- Duration Vector Models
- Applications to Financial Institutions

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Financial institutions and other market participants manage many types of risks

- Interest rate risk: is the risk that the relative value of an interest-bearing asset, such as a loan or a bond, will worsen due to an interest rate increase.

- Credit risk: is the risk of loss due to a debtor's non-payment of a loan or other line of credit (either the principal or interest (coupon) or both).

- Foreign exchange risk: is the risk that the value of an investment will decrease due to moves in the foreign exchange rates .

Financial institutions and other market participants manage many types of risks

- Liquidity risk: arises from situations in which a party interested in trading an asset cannot do it because nobody in the market wants to trade that asset.

- Operational risk: defined as the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events.

Our goal is to understand interest risk management

Interest rate risk comes from movements on the yield curve

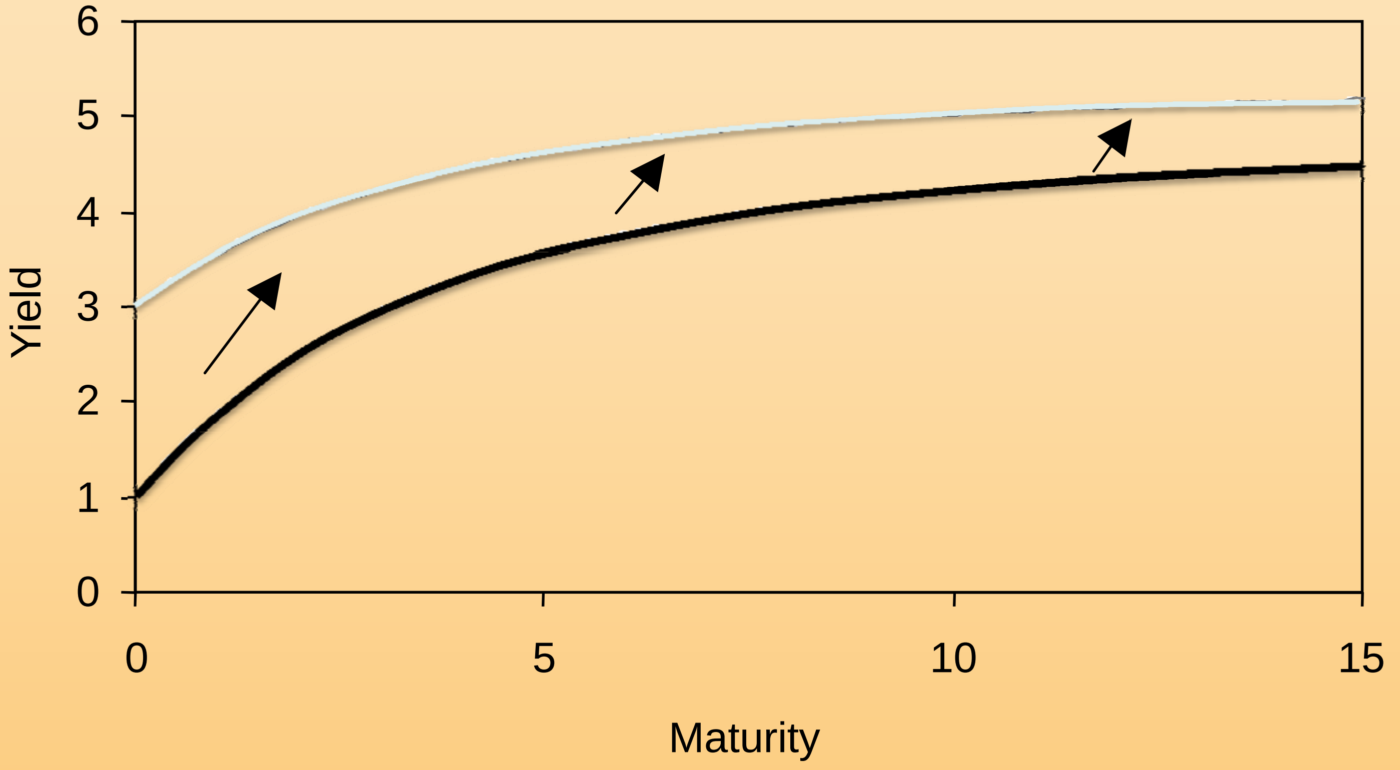


Figure 1.1. Non-parallel yield curve shift

Why interest risk management is important for financial institutions?

- For example, a change in the interest rate will affect the value of for a pension fund's assets in a different amount of its liabilities.

- Other typical example is a commercial bank whose assets mostly consist of fixed and floating rate loans (some with embedded options), while its liabilities consist of deposits in checking, savings, money market accounts, and some debt securities.

How financial institutions manage interest rate risk?

- How do the managers of financial institutions, such as banks, insurance companies, and index bond funds hedge against the effects of non-parallel yield curve shifts?

- How do hedge funds managers design speculative strategies based upon yield curve movements?

- We will address these issues by giving a detailed introduction to the widely used models in the area of interest rate risk management over the past two decades.

Risk management is done with immunization and hedging strategies

- *Interest rate immunization* is a strategy that ensures that a change in interest rates will not affect the value of a portfolio.

- It can be accomplished by several methods, including cash flow matching, duration matching, and volatility and convexity matching. It can also be accomplished by trading in bond forwards, futures, or options.

- If the immunization is incomplete, these strategies are usually called *hedging*. If the immunization is complete, these strategies are usually called *arbitrage*.

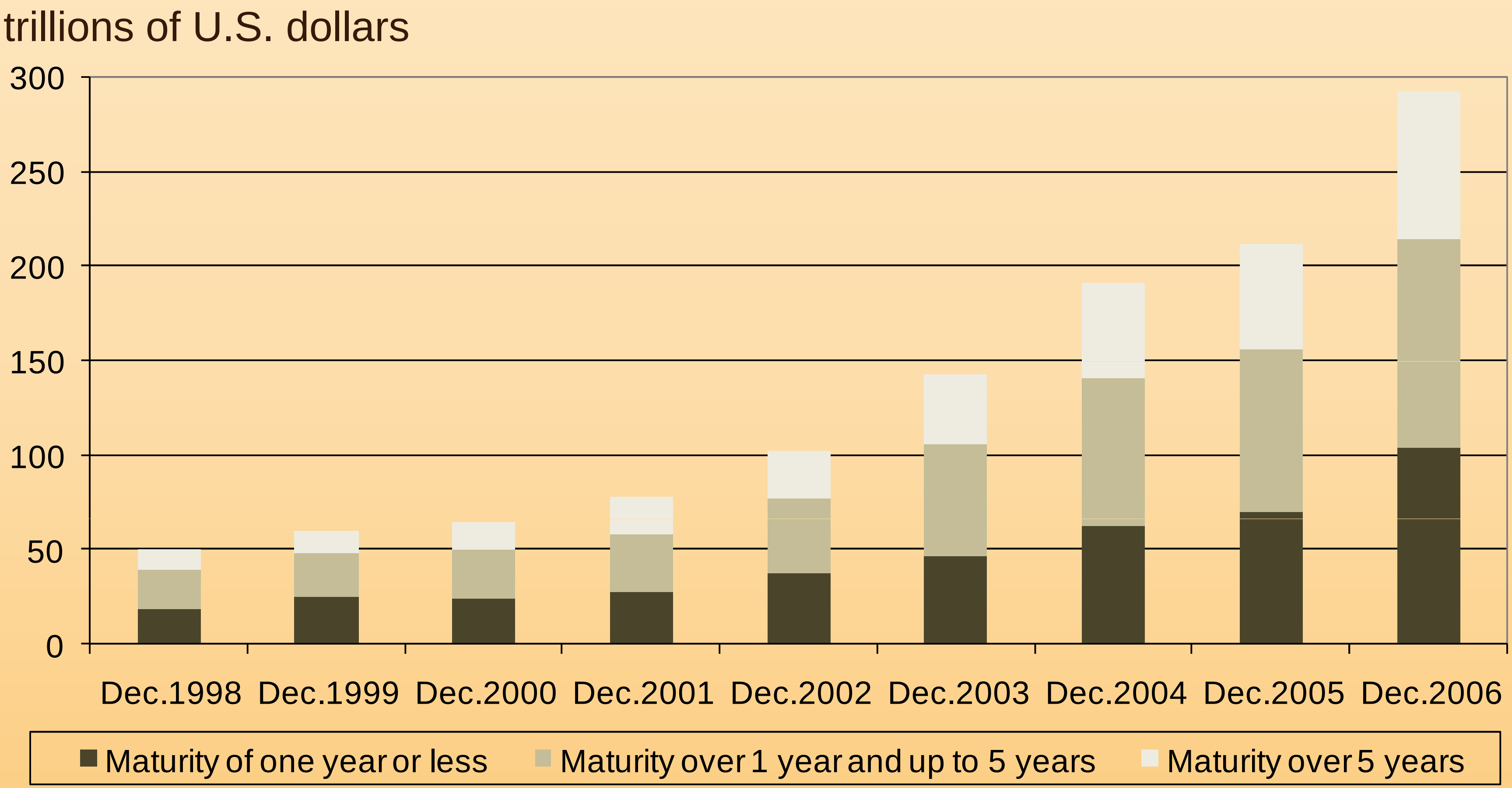
Risk management is a huge market

- Interest rate risks can be hedged using fixed income instruments like a fixed-for-floating interest rate swap.

- The total notional amount of outstanding interest rate derivatives reached $300 Trillion at the end of 2006, $250 trillion in Swaps.

- The explosive growth of interest rate derivatives suggests that managing interest rate risk remains a chief concern for many financial institutions.

Amounts outstanding of OTC single-currency interest rate derivatives



Source: The website of Bank for International Settlements.
http://www.bis.org/statistics/derstats.htm

FixedIncomeRisk.com

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Duration and convexity models

- Average maturity, more popularly known as the *duration* of a security is the most commonly used risk measure for measuring the interest rate risk exposure of the security.

- Consider a bond with cash flows C_t, payable at time t. The bond sells for a price P, and is priced using a term structure of continuously compounded zero-coupon yields given by $y(t)$.

Duration

- The traditional duration model can be used to approximate percentage change in the bond price as follows:

$$\frac{\Delta P}{P} \cong -D\Delta y \qquad (1.1)$$

where,

$$D = \text{Duration} = \sum_{t=t_1}^{t=t_N} t \; w_t$$

and

$$w_t = \left[\frac{C_t}{e^{y(t)\cdot t}} \right] / P$$

Duration

- Duration is given as the weighted average time to maturity of the cash flows, where the weights are defined as the present values of the cash flows divided by the bond price.

- The duration model given in equation (1.1) assumes that the yield curve experiences infinitesimal and parallel shifts.

- The change in the yield Δy, is assumed to be *equal* for all bonds regardless of their coupons and maturities.

Convexity

- Convexity is given as the weighted-average of maturity-squares of a bond, where weights are the present values of the bond's cash flows, given as proportions of bond's price. Convexity can be mathematically expressed as follows:

$$CON = \sum_{t=t_1}^{t=t_N} t^2 w_t \qquad (1.2)$$

$$\frac{\Delta P}{P} \cong -D\Delta y + \frac{1}{2} CON(\Delta y)^2 \qquad (1.3)$$

- Equation (1.3) suggests that for bonds with identical durations, higher convexity is always preferable. Why?

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M-Square

- Duration and Convexity models are based upon the assumption of a parallel shift in the yield curve.

- However, such shifts occur rarely in the bond markets. Even under slight violations of the assumption of parallel yield curve shifts, higher convexity may not be desirable.

M-Square

- An alternative view of convexity is the M-square risk measure, which is based upon a more realistic economic framework.
- It relates convexity to *slope shifts* in the term structure of interest rates.
- This view of convexity was proposed by Fong and Vasicek [1983, 1984] and Fong and Fabozzi [1985] through the introduction of the new risk measure, M-square, which is a linear transformation of convexity.

M-Square

- The M-square of a bond portfolio is given as the weighted average of the squares of the distance between cash flow maturities and the planning horizon of the portfolio.

$$M^2 = \sum_{t=t_1}^{t=t_N} (t - H)^2 \cdot w_t \qquad (1.4)$$

where the weights are defined in equation, and *H* is the planning horizon.

M-Square

- A bond portfolio selected with minimum M-square has cash flows clustered around the planning horizon date and hence, protects the portfolio from immunization risk resulting from non-parallel yield curve shifts.

- The “convexity view” and the “M-square view” have exactly opposite implications for bond risk analysis and portfolio management:
  - Convexity emphasizes the *gain* in the return on a portfolio, against large and parallel shifts in the term structure of interest rates.
  - On the other hand, M-square emphasizes the *risk exposure* of a portfolio due to slope-shifts in the term structure of interest rates.

M-Absolute

- Unlike M-square model, that requires two risk measures for hedging (i.e., both duration and M-square), Nawalkha and Chambers [1996] derive the M-absolute model, which only requires one risk measure for hedging against the non-parallel yield curve shifts.

M-Absolute

- The M-absolute of a bond portfolio is given as the weighted average of the absolute distances between cash flow maturities and the planning horizon of the portfolio:

$$M^A = \sum_{t=t_1}^{t=t_N} \left| t - H \right| \cdot w_t \qquad (1.5)$$

where the weights are defined in equation (1.1), and *H* is the planning horizon.

M-Absolute

- The relative desirability of the duration model or the M-absolute model depends on the nature of term structure shifts expected.
  - If height shifts completely dominate the slope, curvature, and other higher order term structure shifts, then the duration model will outperform the M-absolute model.
  - If, however, slope, curvature, and other higher order shifts are relatively significant – in comparison with the height shifts – then the M-absolute model may outperform the traditional duration model.

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Duration Vector Models

- Though M-square risk measures provide significant enhancement in the immunization performance over the traditional duration model, perfect immunization is not possible except for the trivial case in which the portfolio consists on a zero-coupon bond maturing at the horizon date.

- Further gains in immunization performance have been made possible by the duration vector model, which using a vector of higher-order duration measures immunizes against changes in the shape parameters (i.e., height, slope, curvature, etc.) of the yield curve.

Duration Vector Models

- The immunization constraints of the duration vector model are given by the following equation:

$$D(m) = \sum_{t=t_1}^{t=t_N} t^m \cdot w_t = H^m, \text{ for } m = 1,2,3,\ldots,Q \quad (1.6)$$

where the weights are defined in equation (1.1), and *H* is the planning horizon. About three to five duration vector constraints (i.e., Q = 3 to 5) have shown to almost perfectly immunize against the risk of non-parallel yield curve shifts.

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Applications to Financial Institutions

- The models given above can be applied in a variety of contexts by financial institutions, from designing and executing simple duration-based hedging strategies to the most sophisticated dynamic immunization programs based upon multiple risk measures, with off balance sheet positions in swaps, interest rate options, and interest rate futures.

Applications to Financial Institutions – Example 1 Insurance company

- Consider an insurance company that sells guaranteed investment products to institutional investors and/or individuals (e.g., GICs, etc.).
- To guarantee a high yield over a pre-specified horizon, the insurance company may use high quality AAA rated corporate bonds to design a dynamic immunization strategy, instead of simply investing in a riskless zero-coupon bond (e.g., Treasury STRIPs).
- The extra yield on high quality bonds will compensate for the additional risk introduced by the spread AAA spread changes over the Treasury yield curve changes.

Applications to Financial Institutions – Example 1 Insurance company

- Since the largest portion of the yield changes for high quality corporate bonds are due to changes in the Treasury yield curve, a one to three factor duration vector can be used.

- Though using more risk measures will lead to better immunization performance, doing so will require higher transaction costs and may even require explicit or implicit short positions.

- Hence, the number of risk measures should be carefully selected after running many yield curve scenarios with transactions cost analysis.

Applications to Financial Institutions – Example 2 Commercial bank

- As a second example, consider a commercial bank interested in protecting the value of shareholder equity from interest rate risk.

- The equity duration can be computed using the asset duration and the liability duration.

- An appropriate model that can be used to protect a bank's equity is the M-square model with a pre-specified target equity duration.

Applications to Financial Institutions – Example 2 Commercial bank

- This model does not require that the M-square of the assets be set equal to the M-square of the liabilities, but that the difference between the M-squares of the assets and the liabilities be minimized.

- Since by the nature of their business (lend in long maturity sector, borrow in short maturity sector), banks cannot fully adjust the maturity structure of their assets and liabilities, the M-square model is more suited for a bank, instead of the duration vector model.

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